Sheet 2	Communication Theory II, Third Year Electronics and Communication Department

1- Prove the following: (a) a signal and its Hilbert transform have the same magnitude spectrum, (b) a Hilbert transform produces a phase shift of –90° for all positive frequencies of the input signal and +90° degrees for all negative frequencies, and (c) if $\hat{g}(t)$ is the Hilbert transform of $g(t)$, then the Hilbert transform of $\hat{g}(t)$ is $-g(t)$.

2- Using the Bernoulli distribution, (a) develop an experiment that involves three independent tosses of a fair coin. Irrespective of whether the toss is a head or tail, the probability of every toss is to be conditioned on the results of preceding tosses, (b) display graphically the sequential evolution of the results, (c) find the probability that all the three tosses are head assuming (c-1) a fair coin and (c-2) for some reason the coin is not fair and the prior probability that the toss is a head equal 0.6.

3- A discrete memoryless channel is used to transmit binary data. The channel is discrete in that it is designed to handle discrete messages and it is memoryless in that at any instant of time the channel depends on the channel input only at that time. Owing to the unavoidable presence of noise in the channel, errors are made in the received binary data stream. The channel is symmetric in that the probability of receiving symbol 1 when symbol 0 is sent is the same as the probability of receiving symbol 0 when symbol 1 is sent.

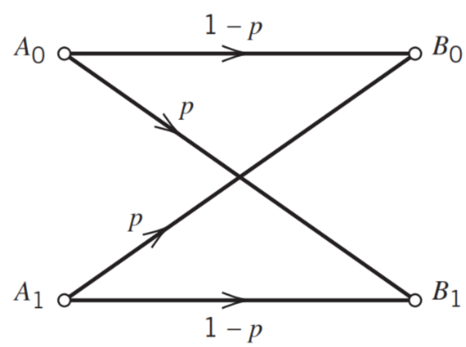
The transmitter sends 0s across the channel with probability p_o and 1s with probability p_1. The receiver occasionally makes random decision errors with probability p; that is, when symbol 0 is sent across the channel, the receiver makes a decision in favor of symbol 1, and vice versa.

Referring to Figure below, determine the following a posteriori probabilities:

a. The conditional probability of sending symbol A_o given that symbol B_o was received.

b. The conditional probability of sending symbol A_1 given that symbol B_1 was received.

Hint: Formulate expressions for the probability of receiving event B_o, and likewise for event B_1



4- The joint probability density function of two random variables X and Y is defined by the two-dimensional uniform distribution

$$f_{X,Y}(x,y) = \begin{cases} c & \text{for } a \le x \le b \text{ and } a \le y \le b \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the scalar c for which the normalization property of a two-dimensional probability density function is satisfied.
(b) Find the marginal probability densities $f_X(x)$ $f_Y(y)$
(c) Find X and Y means and variances
(d) Find the covariance $\mathrm{cov}[XY]$ and the correlation coefficient
(e) Are X and Y orthogonal?
(f) Are X and Y independent?
(g) Are X and Y uncorrelated?

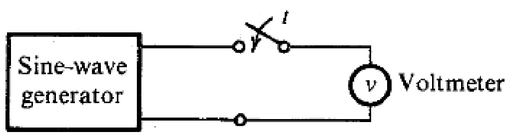
5- The probability density function of a Rayleigh random variable is defined by

$$f_X(x) = \frac{x}{\sigma^2}\exp\left(-\frac{x^2}{2\sigma^2}\right) \qquad \text{for } x \geq 0 \text{ and } \sigma > 0$$

Show that the mean of X is $\mathbb{E}[X] = \sigma\sqrt{\frac{\pi}{2}}$, note that $\int_0^{\infty} x^2 \exp(-ax^2)\,\mathrm{d}x = \frac{1}{4a}\sqrt{\frac{\pi}{a}}$

6- Continuing with the Bernoulli random variable X, find the mean and variance of X.

7- A sinusoid generator output voltage is $A \cos \omega t$. This output is sampled randomly (figure below) The sampled output is an RV x, which can take on any value in the range $(-A, A)$. Determine the mean value $(\overline{x})$ and the mean square value $(\overline{x^2})$ of the sampled output x.



8- Over a certain binary channel, the symbol 0 is transmitted with probability 0.4 and 1 is transmitted with a probability of 0.6. It is given that P(channel error|0)=10^{-6} and P(channel error|1)=10^{-4} where P(channel error|x_i) is the probability of detecting the error given that x_i is transmitted. Determine the error probability of the channel P(channel error)